UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Level

FURTHER MATHEMATICS **9231/02**

Paper 2

May/June 2005

3 hours

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

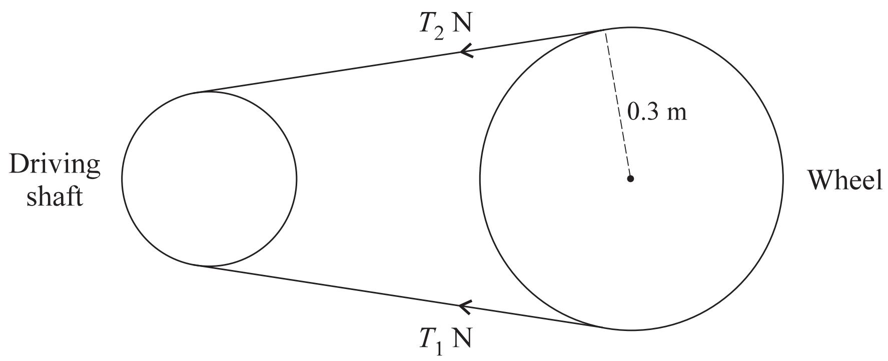
The number of marks is given in brackets [ ] at the end of each question or part question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 A tennis ball of mass 56 g, moving horizontally, is struck by a racquet so that its velocity c
$20\,\mathrm{m\,s^{-1}}$ east to $30\,\mathrm{m\,s^{-1}}$ west. Find the magnitude of the impulse given to the ball by the rac

At a later instant the ball is travelling horizontally at $25\,\mathrm{m\,s^{-1}}$ when it hits the net. This reduce horizontal velocity to zero in 0.2 s. What constant horizontal force does the net exert on the ball? [2

2



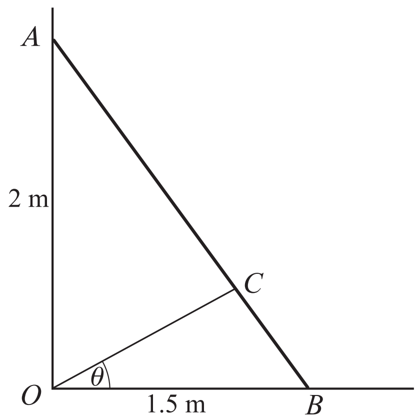
The cylindrical driving shaft of an engine is connected to a wheel of radius 0.3 m by a taut, light, non-slipping and inextensible belt around part of the circumference of each. The moment of inertia of the wheel about its axis is $600\,\mathrm{kg\,m^2}$, and the wheel is free to rotate. The engine maintains a difference in tension between the two straight parts of the belt of 400 N (so that the forces T_1 N and T_2 N, in the diagram, are related by $T_1 - T_2 = 400$). Find the angular speed of the wheel 30 s after starting from rest. [3]

Find also the number of revolutions, correct to 1 decimal place, made by the wheel during this 30 s interval. [3]

3 Three balls, A, B and C, of the same size but of masses $2m$, m and $2m$ respectively, lie at rest in a straight line on a smooth horizontal table. The ball B, which is between A and C, is projected towards C with speed $36u$. The coefficients of restitution between A and B, and between B and C, are each $\frac{3}{4}$. Show that the second collision is the last. [7]

Express the total loss of kinetic energy of the system as a percentage of the original kinetic energy. [2]

4



A uniform ladder AB, of weight W N and length 2.5 m, rests against a smooth vertical wall OA with its foot on smooth horizontal ground OB. The ladder is in a vertical plane perpendicular to the wall. It is kept in position with $OA = 2$ m and $OB = 1.5$ m by a light rope OC joining O to a point C on the ladder, such that angle $COB = \theta$. Show that the tension T N in the rope is given by

$$T = \frac{3W}{8\cos\theta - 6\sin\theta}.$$ [7]

Deduce that $\tan\theta < \frac{4}{3}$. [2]

Find T in terms of W in the case when the rope is perpendicular to the ladder. [2]

5 A simple pendulum is formed by a light string of length l with a small bob B of mass m at one end. The string hangs from a fixed point O at the other end. The string makes an angle θ with the vertical at time t. Write down the equation of motion of B perpendicular to OB, and state the approximation that enables you to treat this as an equation of simple harmonic motion. State the general solution of this equation, and the period of the approximate simple harmonic motion that it represents. [7]

Given that the amplitude of the simple harmonic motion is α, show that the pendulum swings from the vertical to the position where $\theta = \frac{1}{2}\alpha$ in one twelfth of the period. [5]

6 A bulb is tested by switching on and off repeatedly. Each time the bulb is switched on the probability that it fails is $\frac{1}{200}$. Assuming independence, find the probability that the bulb is still burning after it has been switched on 100 times. [2]

The bulb fails when it is switched on for the Nth time. Write down the value of $\mathrm{E}(N)$, and find the greatest value of n such that $\mathrm{P}(N \leqslant n)$ is less than 0.1. [4]

7 Each elephant in a random sample of 10 Indian elephants was weighed and the results are given below.

Mass in tonnes: 4.2, 4.8, 5.1, 5.3, 5.3, 5.5, 5.7, 5.9, 6.2, 6.4.

Find a 98% confidence interval for the mean mass of an Indian elephant, and state an assumption needed for your calculation to be valid. [8]

8 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{\theta}e^{-\frac{x}{\theta}} & x \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where θ is a positive constant. Show that the mean of X is θ, and determine which is the greater, $P(X < \theta)$ or $P(X > \theta)$. [4]

Given that a and b are positive, show that $P(X > a + b \mid X > a) = P(X > b)$. [3]

Explain the meaning of this equation in the case when X is the number of days after treatment for a particular disease when a patient needs further treatment. [2]

9 Lunch expenses claimed by stockbrokers and bankers are being compared, and random samples give the following figures in suitable units.

$$\begin{array}{rlll} \text{Stockbrokers:} & n_1 = 8, & \bar{x}_1 = 15.5, & s_1^2 = 5.29, \\ \text{Bankers:} & n_2 = 12, & \bar{x}_2 = 12.8, & s_2^2 = 15.21, \end{array}$$

where n_1, n_2 are the sample sizes, $\bar{x}_1$, $\bar{x}_2$ are the sample means and s_1^2, s_2^2 are the unbiased estimates of the population variances. Assuming that the two populations have normal distributions with means μ_1 and μ_2 respectively, and equal variance,

(i) test at the 5% level of significance the hypothesis that, on average, stockbrokers claim more than bankers, [7]

(ii) obtain a 95% confidence interval for $\mu_1 - \mu_2$, giving 2 decimal places in your answer. [3]

10 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 1 + x & -1 \leqslant x \leqslant 0, \\ 1 - x & 0 < x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Show that its distribution function is

$$F(x) = \begin{cases} 0 & x < -1, \\ \frac{1}{2}(1 + x)^2 & -1 \leqslant x \leqslant 0, \\ 1 - \frac{1}{2}(1 - x)^2 & 0 < x \leqslant 1, \\ 1 & x > 1. \end{cases}$$ [3]

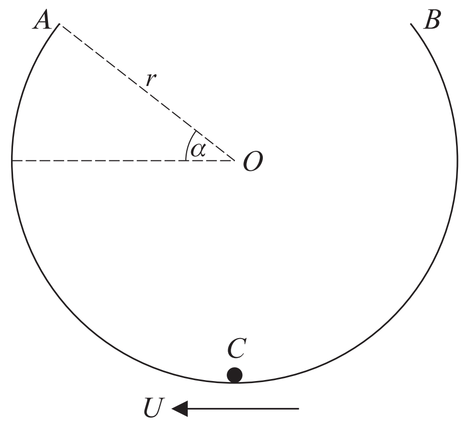
The random variable Y is defined by $Y = X^2$. Show that

$$P(Y \leqslant y) = 2\sqrt{y} - y, \quad 0 \leqslant y \leqslant 1.$$ [3]

Find $E(Y)$. [4]

11 Answer only **one** of the following two alternatives.

EITHER



A smooth metal strip with ends A and B is bent into a major arc of a circle with centre O and radius r. The strip is fixed in a vertical plane with AB horizontal and at a height $r \sin \alpha$ above O, where $0 < \alpha < \frac{1}{2}\pi$ (see diagram). A particle is projected from the lowest point C with speed U so that it travels along the inside of the strip, reaching A with speed V. The particle then moves under gravity, and arrives at B. Air resistance may be neglected. Show that

(i) $V^2 = gr \operatorname{cosec} \alpha$, [4]

(ii) $U^2 = gr(\operatorname{cosec} \alpha + 2 + 2 \sin \alpha)$. [3]

The mass of the particle is m and the normal reaction between the strip and the particle is R. Show that the greatest value of R is $mg(\operatorname{cosec} \alpha + 3 + 2 \sin \alpha)$, and find the least value of R. [7]

OR

A British tourist in a European town noted that 6 items carried a price in pounds (£x) and also a price in euros (€y). The table shows the observations.

x	6	8	15	18	23	27
y	7	12	20	28	35	36

Find the least squares regression line of y on x. Also find r_1, the product moment correlation coefficient of x and y. [6]

Use this regression line to predict the price in euros of an article labelled £30, commenting on the advisability of this procedure. [4]

When the observations were made the official rate of exchange for £1 was €1.62. Values of z are such that €z is the equivalent of £x at this rate, so that $z = 1.62x$. The product moment correlation coefficient of y and z is r_2. Without further calculation, show that $r_2 = r_1$. [You should start from a formula for the product moment correlation coefficient given in the List of Formulae, MF 10.] [4]

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